Atmospheric Turbulence Introduction

Dr. Arjuna Flenner
Collaborators: Dr. Arun Majumdar, David O’Connor, Larry Peterson

1NAWCWD - China Lake
Physics and Computational Sciences
Outline

1. Basics of Turbulence
2. Image Models
3. Math Modeling of Turbulence
Random inhomogeneities of the refractive index in air creates amplitude and phase variations of incoming waves.

- Large and small length scales in an isotropic stretch of turbulence. Turbulence is generated by
  - Convection: Temperature gradients in the atmosphere.
  - Wind: Different layers of the atmosphere and wind motion over objects.
- Different turbulent “regions” exist due to changing atmospheric conditions
  - The turbulence strength and length scales are different in each region.
  - The region size can be on the order of meters to kilometers, and depends on the environments.
  - A big problem is when we need to image across several such regions. A uniform assumption on the turbulence parameters is not valid.
  - This phenomena is sometimes called “deep turbulence”.

Scales in Komologorov Turbulence
Fried Parameter and Scales

- The largest length scale is characterized by
  \[ k_0 = \frac{2\pi}{L_0}, \]
  where \( L_0 \) is a largest length scale of the turbulence.
- This large motion then transfers energy to smaller length scales until the viscous dissipation scale \( L_\nu \ll L_0 \)
- Due to a scaling argument, this means energy per mode \( k \) per gram of gas must satisfy the relationship
  \[ E(k) = C\epsilon^{2/3}k^{-5/3}, \]
  where \( \epsilon \) is the energy input rate per gram of gas.
- The time scale for eddy motion is
  \[ \tau \propto \epsilon^{-1/3}k^{-2/3}. \]
- This leads to about a millisecond as the time scale for turbulence. We would need to sample video at kilohertz rates to capture stationary samples.
Structure Function

- The position dependent gas velocity $v(x)$ plays a critical role in turbulence, thus the structure function

$$D_v(x_1 - x_2) = \mathbb{E}[|v(x_1) - v(x_2)|^2]$$

is central to a statistical analysis of turbulence.

- Scaling arguments based on the energy gives a structure function for the index of refraction

$$n(x, t) \sim 1 + .00029 \frac{\rho(x, t)}{1.3 \times 10^{-3} gcm^{-3}}$$

$$D_n(x_1, x_2) = C_n^2|x_2 - x_1|^{2/3}.$$ 

- The function $C_n^2$ is a function of position. Define the Fried Parameter as

$$r_0 = \left( .423 k^2 \int_{\text{path}} C_n^2(x)\,dx \right)^{-3/5}.$$ 

- This is the length scale on which the phase error is on the order of one wavelength.
Nonisoplanatic

Let $r$ be a path length imaged through the atmosphere, and $\theta$ be an angular separation between two sources. Then the resolving angle $\theta$ due to turbulence is given by

$$\bar{\sigma}^2 = \left( \frac{\theta}{\theta_0} \right)^{5/3}$$

$$\theta_0 = \left( 2.915 \, k^2 \, \theta^{5/3} \, \mu_{5/3} \right)^{-3/5}$$

$$\mu_{5/3} = \int_{\text{path}} C_n^2 \, x^{5/3} \, dx.$$

For video processing, i.e. short exposure time, this characterizes the angular separation where the point spread function changes.

For deep turbulence, this angular separation is less than one pixel. The point spread function changes on a length scale of less than a pixel.
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Important Image Characteristics - Metrics

There is a goal in the image processing system, and several parameters are important to restore.

- Straight Lines
- Object boundaries
- Edges
- Object Centers
Image Model - ROF

The Rudin-Osher-Fatemi (ROF) model preserves edges

- Let $u_0$ be an input image, then the ROF model for noise removal
  minimizes

  $$
  \hat{u} = \arg\min_u \int |\nabla u| \, dx + \frac{\lambda}{2} \int (u - u_0)^2 \, dx.
  $$

- The total variation regularization term preserves edges.
- In turbulence models, the fidelity term is modified using an unknown
  function $K(\cdot)$ that may also need to be learned

  $$
  (\hat{u}, \hat{K}) = \arg\min_{u,K} \int |\nabla u| \, dx + \frac{\lambda}{2} \int (K(u) - u_0)^2 \, dx + G(K).
  $$
The Mumford-Shah Model also preserves edges

- Let \( u_0 \) be an input image and \( \Gamma \) be an edge set, then the Mumford-Shah model considers

\[
\hat{u} = \arg\min_{u} \mathcal{H}(\Gamma) + \beta \int_{\Omega \setminus \Gamma} |\nabla u|^2 \, dx + \lambda \int_{\Omega} (K(u) - u_0)^2 \, dx.
\]

- The \( \mathcal{H}(\Gamma) \) term is the 1-D Hausdorff measure.
- This energy can be difficult to approximate to minimize.
  - One approximate method has been the Ginzburg-Landau energy functional.
We want to find a dictionary \( A = (a_1, a_2, \ldots, a_N) \) to represent each \( M \times M \) image patch according to the following optimization problem:

\[
\min_{A,(x_i)} \sum_{i=1}^{M} \|x_i\|_0 \text{ subject to } \|y_i - Ax_i\|_2 \leq \epsilon.
\]

Description: Each of the image patches are given as a sparse representation of \( x_i \) over a unknown dictionary \( A \).

Can a learn dictionary compensate for the nonisoplanatic assumption?
Lucky Imaging

- Goal is to find high quality images that have little turbulence effects.
- Probability of a Lucky image is

\[ P \propto 5.6 \exp[-0.1557 \times (D/r_0)^2] \]

where \( r_0 \) is the Freid parameter and \( D \) is the telescope diameter. We assume Kolmogorov turbulence.

- Wait for some lucky images and then use the low turbulence images to recreate the scene.
- Can we count on getting enough Lucky images? This is a time scale question.
- Lucky imaging is unreliable in time critical situations.
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Composition of Functions model

- Turbulence creates a distortion of an image from a “true” turbulence free image.
- Model the turbulence using a function $F(u)$. We want to “invert” this function.
- One approximation is to model $F(u)$ as the composition of a “blur” and an image distortion.

$$F(u) = G(K(u))$$

where $G$ is image distortion and $K$ is a blur.
- This suggest we 1) remove image distortion then 2) remove the blur.
  - How are we going to achieve this goal?
  - Can we achieve this goal with one image?
  - Is this assumption appropriate for nonisoplanatic images?
Video Sequence

- Most analysis needs to be done on a video sequence.
- Time constraint is a major issue - can processing be done at near video rates (30 Hz typical and 60Hz or 120 Hz possible).
- Can we exploit the video sequence information
  - averaging operation to find a blurred pixel
  - optical flow to remove distortion
Conclusion

- Turbulence is a difficult topic in image processing.
- The assumption of a linear blur kernel on small regions may not be valid.
  - How do we handle “deep” turbulence?
  - What can more sophisticated turbulence models tell us?
  - What can be done about nonisoplanatic turbulence?
- New models and methods may be needed to solve the turbulence problem.